EXPLANATION OF EXAMPLE CODE

To illustrate a simple Gibbs sampler in MATLAB and R, consider a data set $x = (x_1, \ldots, x_n)$ which each x_i is distributed as:

$$x_i \overset{iid}{\sim} N(\mu, \sigma^2)$$

for $i = 1, \ldots, n$ where μ and σ^2 are unknown parameters. The goal of this basic analysis is to estimate μ and σ^2. Assume the prior distributions for μ and σ^2 are:

$$\mu \sim N(m, s^2)$$
$$\sigma^{-2} = \phi \sim Gam(a, b)$$

where m, s^2, a, and b, are known hyperparameters specified by the researcher. Given this prior structure, the joint posterior distribution $p(\mu, \sigma^2|x)$ does not have closed form. Therefore, in order to conduct inference on μ and σ^2, we need to obtain samples $(\mu, \sigma^2)_j$ from $p(\mu, \sigma^2|x)$. To do so we can use the Gibbs sampler. Th Gibbs sampling algorithm for this example is:

(1) Sample μ from $p(\mu|\sigma^2, x)$
(2) Sample σ^2 from $p(\sigma^2|\mu, x)$

where $p(\mu|\sigma^2, x)$ is the "complete conditional" distibution of μ and $p(\sigma^2|\mu, x)$ is the complete conditional distribution of σ^2.

Under the prior distributions mentioned above and employing Bayes theorem, it can be shown that the complete conditional for μ is normal and the complete conditional for σ^2 is inverse gamma. Specifically,

$$\mu|\sigma^2, x \sim N(m^*, s_*^2)$$
$$\sigma^{-2}|\mu, x = \phi|\mu, x \sim Gam(a^*, b^*)$$

where,

$$m^* = \frac{\frac{1}{s^2}m + \frac{n}{\sigma^2}\bar{x}}{\frac{1}{s^2} + \frac{n}{\sigma^2}}$$
$$s_*^2 = \frac{1}{\frac{1}{s^2} + \frac{n}{\sigma^2}}$$
$$a^* = a + \frac{n}{2}$$
$$b^* = \frac{\sum_{i=1}^{n}(y_i - \mu)^2}{2} + b.$$

Given these complete conditional distributions, the Gibbs sampling algorithm will then proceed as follows:

(1) Sample μ from $N(\mu^*, s_*^2)$
(2) Sample σ^2 by sampling ϕ from $Gam(a^*, b^*)$ and setting $\sigma^2 = 1/\phi$.

The example code is this algorithm written in MATLAB and R. The code also provides examples of how to write data to a file, read data from a file, and plot some simple figures.

If you need help understanding the functions used within the code, type help *functionname* in MATLAB or help(*functionname*) in R or ask your neighbor.